



Understanding Accuracy-fairness Trade-offs in Re-ranking Through Elasticity in Economics

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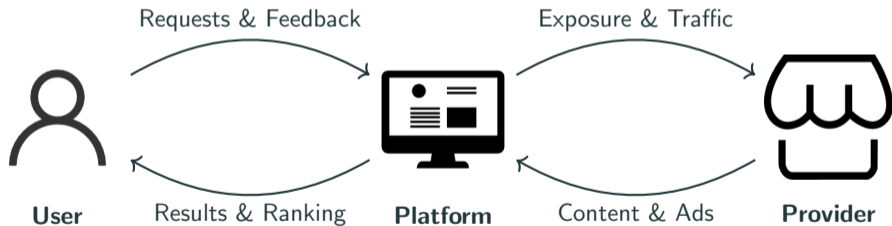
- 1 Background: What is accuracy-fairness trade-off in re-ranking?
- 2 Motivation: Elastic perspective on such trade-offs
- 3 Methods: Elasticity-inspired evaluation and ranking framework
- 4 Conclusion

**Background: What is
accuracy-fairness trade-off in
re-ranking?**

What is provider fairness in re-ranking?

Stakeholder Interactions in IR

- **User**, **Platform**, and **Provider** form a dynamic ecosystem
- Each stakeholder has different goals and influences the system



- Fair exposure for providers is essential in RS



Figure 1: Providers require a certain degree of user engagement to remain viable.

Examples

- Assuming N users (u_1, u_2, \dots, u_N)
- Assuming M items/documents (i_1, i_2, \dots, i_M).
- Re-ranking systems can only adjust the **ranking score matrix** X

	u_1	u_2	\dots	u_N
i_1	• - 0.8	• - 0.6	• - 0.5	
i_2	• - 0.7	• - 0.7	• - 0.6	
i_3	• - 0.5			• - 0.7
\dots		• - 0.8		• - 0.5
i_M			• - 0.4	• - 0.2

Provider 1
utility = 1.95

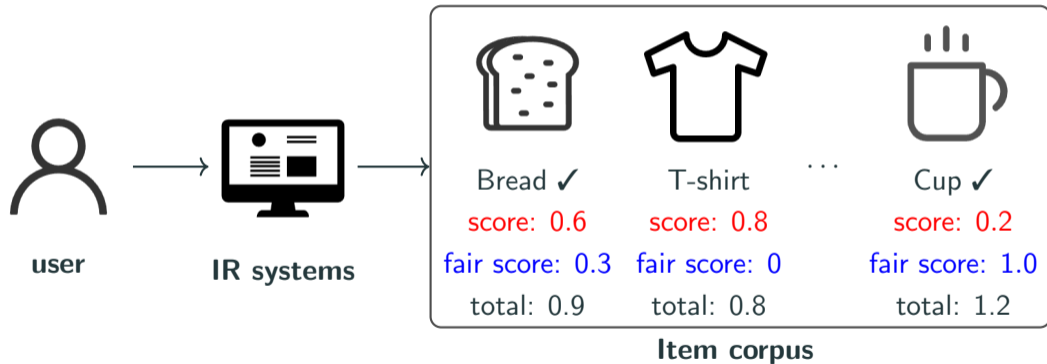
Provider m
utility = 0.6

provider fairness

Fairness-accuracy trade-offs in re-ranking

Previous Fair-aware Re-ranking Methods

They aim to increase the exposure of certain providers: **Through fairness score!**



Fairness-accuracy Trade-offs

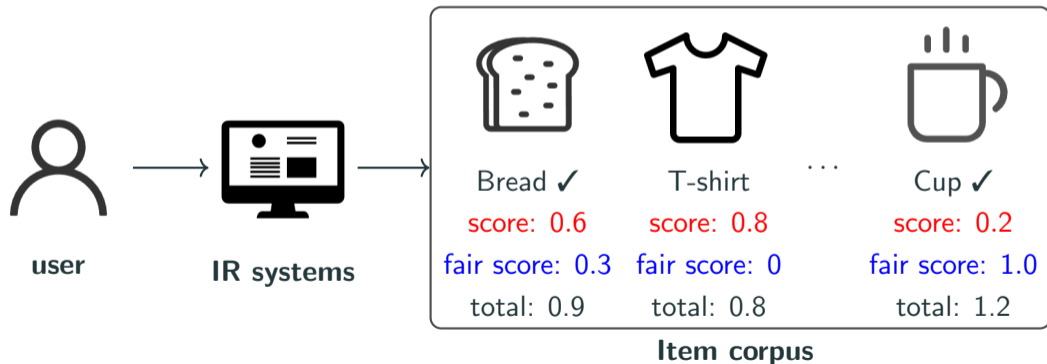
Fairness for providers will influence **user's utilities** by changing the ranking list (size=2):

- W/o fairness: total score will be $0.6 + 0.8 = 1.4$ (Bread and T-shirt)

Fairness-accuracy Trade-offs

Fairness for providers will influence **user's utilities** by changing the ranking list (size=2):

- W/o fairness: total score will be $0.6 + 0.8 = 1.4$ (Bread and T-shirt)
- With fairness: total score will be $0.6 + 0.2 = 0.8$ (Bread and Cup)



**Motivation: Elastic perspective on
such trade-offs**

Elasticities in Economics

Definition:

Elasticity measures the responsiveness of one economic variable to a change in another.

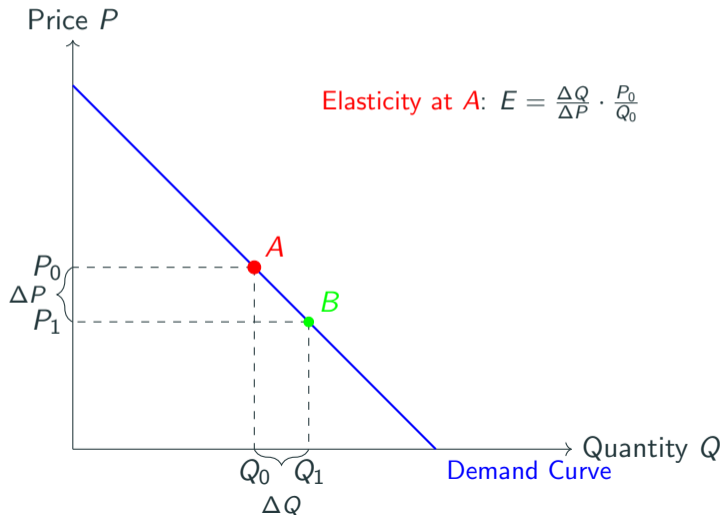
Price Elasticity of Demand:

$$E = \frac{\% \text{ Change in Quantity}}{\% \text{ Change in Price}} = \frac{\partial Q/Q}{\partial P/P},$$

where Q is the purchase quantity of an item and P is the item price.

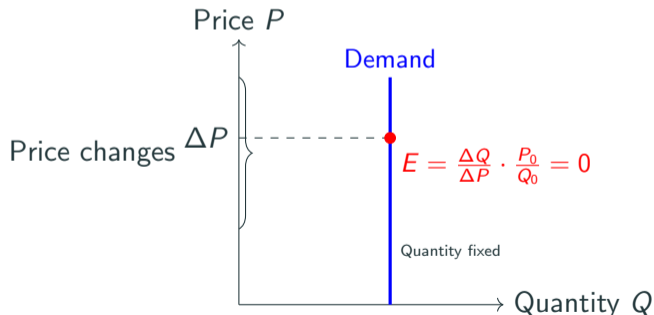
Examples

- Elasticity geographic explanation:



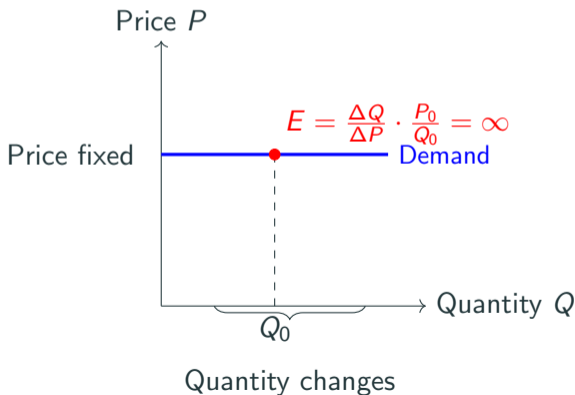
Elasticity Applications

- The **taxation transfer problem** is decided by the elasticity!
- If we impose a tax on providers selling **necessities (elasticity = 0)**, providers can **pass the tax on to users** by raising prices.



Elasticity Applications

- The **taxation transfer problem** is decided by the elasticity!
- If we impose a tax on providers selling **luxury (elasticity = ∞)**, providers cannot raise the price and afford the tax on their own.



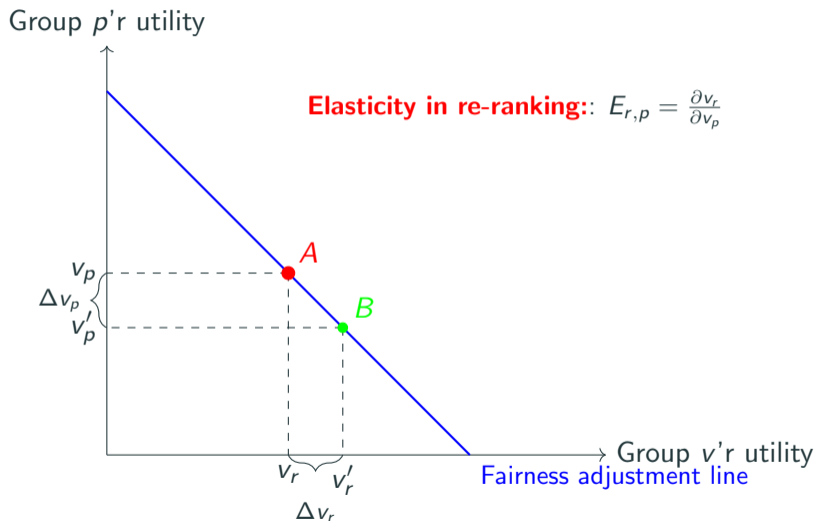
Elasticity for Fairness-Accuracy Tradeoffs

Accuracy-fairness trade-off problems can be regarded as the taxation transfer problem in economics!

Table 1: Correspondence between taxation elements in economics and fair re-ranking.

Economics	Fair re-ranking
Consumer (buy product)	Users \mathcal{U} (click items)
Supplier (sell product)	Item groups \mathcal{G} (provide items)
Commodity tax	Fairness constraint
Tax subsidies for the poor	Increase ranking score for the poor
Selling price (tax objective)	Ranking scores (fairness objective)
Elasticity on price E_e	Elasticity on utilities of item group $E_{r,p}$

Alignment



Theoretical Analysis

Theory proves that fairness-accuracy trade-offs are related to elasticity!

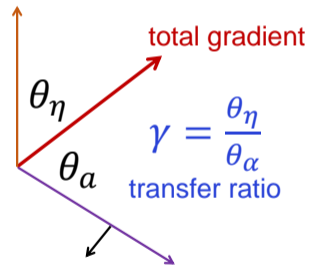
Theorem

The transfer ratio between fairness (commodity tax on groups) and accuracy (commodity tax transferred to users) is:

$$\gamma = \frac{\langle \nabla_{\mathbf{v}} L, \eta \rangle}{\langle \nabla_{\mathbf{v}} L, \alpha \rangle} = 1 - \frac{1}{1 + k(E_{r,p})},$$

$$k(E_{r,p}) = \frac{\sum_{p \in \mathcal{G}} \sum_{r \neq p} \mathbf{v}_p^{1-|t|} E_{r,p}}{\sum_{p \in \mathcal{G}} \mathbf{v}_p^{1-|t|}}$$

η : fair gradient
(commodity tax on item)



α : accuracy gradient
(commodity tax on users)

Benefits for Such Alignments

1. Such alignments inspire us to design a better evaluation framework.
2. Such alignments inspire us to design a more effective and efficient fair-aware re-ranking algorithm.

Methods: Elasticity-inspired evaluation and ranking framework

According to elasticity, we can design a unified fairness evaluation function:

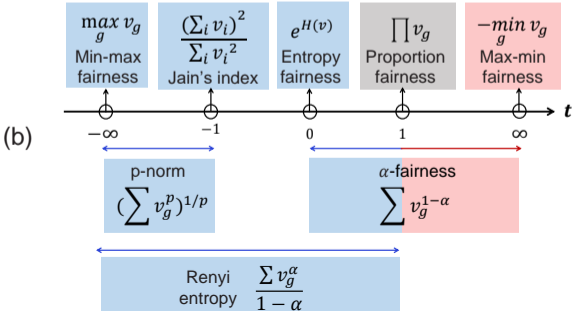
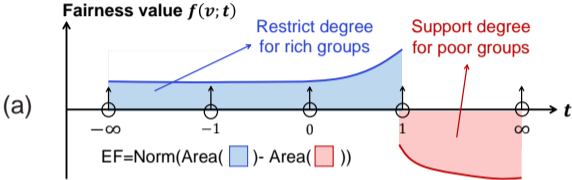
$$f(\mathbf{v}; t) = \text{sign}(1 - t) \left(\sum_{g=1}^{|\mathcal{G}|} \bar{\mathbf{v}}_g^{1-t} \right)^{\frac{1}{t}},$$

Evaluation: EF-curve

Each elastic value t can be aligned with one specific fairness metric

$$EF = \int_{1-M}^{1+M} \frac{f(\mathbf{v}; t)}{Z} dt, \tag{1}$$

where $Z = 2M|\mathcal{G}|$ is the normalized factor.



The additional distance $d(g(i), a)$ is computed through the curve distance:

$$d(g(i), a) = \int_{v_{g(i)}}^{v_a} \sqrt{1 + \left(\frac{\partial v_a}{\partial x}\right)^2} dx \approx (1-t) \mathbf{v}_{g(i)}^{-t} (\mathbf{v}_a^{1-t} - \mathbf{v}_{g(i)}^{1-t}), \quad (2)$$

The $d(g(i), a)$ can be viewed as the fairness weight for the original ranking scores!

- **Metric:**

$$\text{Accuracy: NDCG@K} = \frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \frac{\sum_{i \in \mathbf{L}_K^F(u)} s_{u,i} / \log(\text{rank}_i^F + 1)}{\sum_{i \in \mathbf{L}_K(u)} s_{u,i} / \log(\text{rank}_i + 1)},$$

$$\text{Fairness: EF} = \int_{1-M}^{1+M} \frac{f(\mathbf{v}; t)}{Z} dt,$$

- **Dataset:** Steam, Amazon-Digital-Music and Yelp

Experiments

Pareto dominated all the baselines!

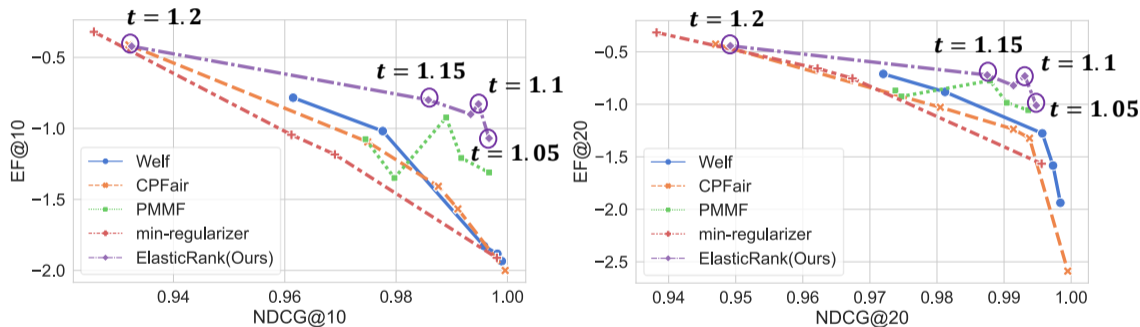


Figure 2: Pareto frontier with different size K under Steam.

Conclusion

- This research frames the re-ranking accuracy-fairness trade-off as a **commodity taxation transfer problem**.

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- Inspired by elasticity theory, we propose a novel evaluation framework **EF-Curve** and algorithm **ElasticRank**

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- Inspired by elasticity theory, we propose a novel evaluation framework **EF-Curve** and algorithm **ElasticRank**
- Rigorous empirical evaluation on three publicly available ranking datasets demonstrates that ElasticRank consistently surpasses state-of-the-art baselines.

Q&A



Paper

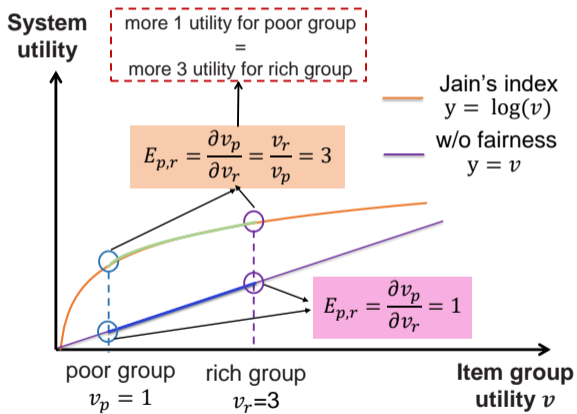
Code

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(I am looking for postdoc opportunities)

Elasticities in re-ranking: Examples



Taxation Transfer Problems as Trade-offs

